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Signaling bargaining power: Strategic delay versus restricted offers[⋆]

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Summary. I study the first-round separating equilibrium of a buyer-seller bargaining game, extended to allow for asymmetric information, strategically delayed offers and offers restricted to a portion of the good. When bargaining is over a consumption good, in equilibrium the "strong" buyer uses a restricted offer if his optimal consumption path is conservative relative to the "weak" buyer. A pure restricted offer may even be a costless, efficient signal. When the good is durable, a pure strategic delay is involved in signaling a strong bargaining position if the discount factor is high.

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1 Introduction

Signaling bargaining power is fundamental in bilateral bargaining with incomplete information. Schelling (1956) pioneered a literature that examines various instruments that can be used for this purpose, ranging from the use of commitment devices (adopting a restrictive agenda or delegating authority to a tough agent) to the strategic use of delay. However, little is known about the *optimal* use of such instruments. In particular, does the nature of the object of potential exchange, whether it is durable or a consumption good, affect the best way of signaling bargaining power?

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A well-known signaling strategy in bargaining is what Admati and Perry (1987) have termed *strategic delay*. A strong bargainer can leave the negotiation table for a sufficiently long period of time in order to convey a clear-cut message about his tough bargaining position. Bargaining power can also be signaled through *restricted offers* whose acceptance will still leave some matters to be resolved later. Bac and Raff (1996) have shown that such signals do indeed exist when two substitutable issues are to be negotiated and offers that do not propose a settlement of both issues simultaneously are restricted to just one issue.[1]

It is natural to expect the best signaling mode to depend on the nature of the object of potential exchange, more precisely, on whether the object is a consumption good (a stock that has to be depleted in order to derive a benefit, like an exhaustible resource or a pie) or a durable good like a piece of land, promising a flow of services to the buyer. If the object is not divisible, restricted offers cannot be used and the strategic delay is determined by the differential valuations of buyer types, as shown in Admati and Perry. But when the object is divisible and restricted offers are allowed, relative preferences for dated portions of the object and hence the nature of the object may matter. Bac and Raff consider restricted offers but rule out the use of delay; none of these papers distinguishes between durable and consumption goods.

I present a model in which a buyer whose valuation, high or low, is private knowledge negotiates with a seller by alternating offers over the price of a divisible object with a potentially infinite horizon. The bargaining outcome may involve delayed and/or partial agreements. The solution concept is sequential equilibrium refined through Admati and Perry's (1987) restriction on off-the-equilibrium-path beliefs. The analysis concentrates on *first-round separating sequential equilibria* where the strong buyer signals his type at minimum cost: the weak buyer immediately makes his complete information unrestricted offer and the seller accepts, while the strong buyer uses a signaling instrument (delay and/or restricted offer). The seller, convinced that she is facing a strong buyer, accepts. If the strong buyer's offer is restricted, the seller makes an offer in the next bargaining round, which is accepted and so ends the game.

I show that the strong buyer's equilibrium signaling strategy differs according to whether the object is a consumption good or a durable good. The consumption-smoothing motive increases the signaling value of restricted offers, and a pure restricted offer (no delay) is used if the strong buyer's optimal consumption path is sufficiently more conservative than the weak buyer. When the strong buyer uses a delay-restricted offer mixture, he distorts his optimal consumption path. This signaling strategy does not degenerate as the time interval between two successive rounds of bargaining goes to zero. An interesting implication of these findings is that restricted offers are more likely to be observed in bargaining

[1] Schelling's observation that the bargaining agenda is not neutral to the outcome has been shown formally, by Fershtman (1990) in an alternating-offers bargaining model and by Herrero (1989) through a combination of strategic and axiomatic approaches. Kalai (1977) shows that the proportional solution is the only agenda- invariant solution. Non-neutrality of the outcome to the agenda can be used as a signaling device. See Busch and Horstmann (1997) for this point.

environments where buyer types are more heterogeneous in terms of their optimal consumption patterns. In the case of a durable good, a restricted offer is a relatively costly signaling instrument. In Sect. 4, I show that when the discount factor is almost equal to one (i.e., in frictionless bargaining) the strong buyer will use pure strategic delay unless he is ”relatively and sufficiently” satiated, a condition which roughly requires that the strong buyer's marginal per-period utility be sufficiently close to zero when he acquires the entire object. Since a vanishing marginal valuation is not likely to obtain in most applications, offer restrictions have less signaling value in bargaining over durable goods.

2 Bargaining over a consumption good under incomplete information

There are two players, a seller S who owns one unit of a perfectly divisible pie, and a buyer B. It is common knowledge that the seller's valuation is zero, whereas the buyer's valuation V^b is private knowledge. I assume two types of buyer, a low-valuation ($b = L$) and a high-valuation ($b = H$) buyer. The low- (high-)valuation buyer is denoted $B_L(B_H)$ The seller's prior assessment $\pi_{-1} \in (0, 1)$ that $B = B_H$ is common knowledge. The pie can be stored and its portions be consumed in subsequent periods. The portion c_t consumed in period t yields the buyer of type b the utility $u_b(c_t)$. I assume the following.

(A1) The function $u_b : [0, 1] \to [0, \bar{b}]$ *where* $b = H, L$ *is continuous and increasing, with* $u_b(0) = 0$. *Moreover,* $u_L(c)$ *is concave,* $u_H(c) > u_L(c)$ *for all* $c \in (0, 1]$, *and* $u_b'(c)$ *is bounded from above.*

Concavity of $u_L(c)$ may provide a consumption-smoothing motive, hence the total valuation $V^L(1)$ of B_L may be higher than $u_L(1)$. Nothing is assumed in (A1) about the curvature of $u_H(c)$ except that it be continuous, increasing and lie above $u_L(c)$ for all c.

The players alternate in making offers until they reach an agreement on the whole pie. The active player can *delay* his/her offer and make a *restricted offer*, that is, an offer concerning only a portion X of the pie the seller retains. An offer is thus a price-portion pair (P, X) where P is the price proposed for the portion $0 \leq X \leq 1$ of the pie. The game starts at time zero where the active player is the buyer. *One round of bargaining* is a triplet $\{\Gamma, (P, X), \text{response}\}$ where Γ is the delay chosen by the active player. The passive player gives a response $\in \{\text{Yes}, \text{No}\}$ to the offer (P, X). An *agreement* is an accepted offer, denoted $(P_n, X_n)^A$ and an *outcome* is a collection of dated agreements $\{t_n, (P_n, X_n)^A\}$ such that $\sum X_n \leq 1$. A (pure) *strategy profile* is denoted $\sigma = (\sigma_S, \sigma_L, \sigma_H)$.

Let δ denote the common discount factor. The seller's payoff from the outcome $\{t_1, (P_1, X_1)^A\} \wedge \ldots \wedge \{t_N, (P_N, X_N)^A\}$ is $V_S = \sum_{n=1}^{N} \delta^{t_n} P_n$. The buyer's payoff is relatively complex to derive because he may simultaneously be engaged in two activities, consumption and bargaining.[2] Let us now abstract from the bargaining problem to derive the buyer's valuation $V^b(X)$ of the pie of size $X \leq 1$. This is done by solving the following problem

$$\text{(MAX)}: \qquad V^b(X) = \max_{c_t} \left\{ \sum_{t=0}^{\infty} \delta^t u_b(c_t) \right\} \text{ subject to } \sum_{t=0}^{\infty} c_t = X \ .$$

Let $\{c_0^b, c_1^b, \dots, c_T^b\}$ be the solution to the problem MAX, where T is the terminal consumption period.[3] The optimal consumption path satisfies the feasibility constraint and $u_b'(c_t^b) = \delta u_b'(c_{t+1}^b)$ for $0 \leq t < T$, hence exhibits declining consumption over time. A precise distinction can now be made between the two types of the buyer: B_H has a higher valuation than B_L if $V^H(X) > V^L(X)$ for all $X \in (0, 1]$.

Because active players may delay their offers, bargaining rounds may be longer than consumption periods. I will assume that consumption takes place at the beginning of consumption periods, thus, if the buyer has a pie portion Z at time τ and an agreement is reached at time $t \in (\tau, \tau + 1]$ on a portion X_n, the buyer can consume no sooner than date $\tau + 1$. If the buyer has no pie stock at the beginning of the consumption period $[\tau, \tau + 1)$ and an agreement $\{t, (P_n, X_n)^A\}$ is reached at time $t \in [\tau, \tau + 1]$, consumption can resume at time t, re-synchronizing consumption and bargaining time. The buyer's payoff from an outcome $\rho = \{t_1, (X_1, P_1)^A\} \wedge \dots \wedge \{t_N, (X_N, P_N)^A\}$ can be derived recursively, as follows. Let the buyer have a pie of size Z_N at consumption period τ_N, let his consumption be $c_{\tau_N}^b \leq Z_N$, and suppose that the last agreement terminating the bargaining game is reached at time $t_N \in (\tau_N, \tau_{N+1}]$ on X_N. The buyer's discounted payoff at time t_N is $U_N^b = \delta^{\tau_N + 1 - t_N} V^b(X_N + Z_N - c_{\tau_N}^b) - P_N$ if $Z_N > 0$, and $U_N^b = V^b(X_N) - P_N$ if $Z_N = 0$. One can similarly define the buyer's discounted payoff at agreement dates $t_{N-1}, t_{N-2}, \dots, t_1$.

I introduce below a tie-breaking assumption (A2):

(A2) *If the game has two equilibria that generate exactly the same payoff profile, then the players play the equilibrium involving fewer offers.*

Under complete information, the bargaining game described above generates a unique pair of subgame perfect equilibrium (SPE) payoffs. These are

$$U_0^b = \frac{V^b(1)}{1 + \delta} \quad \text{and} \quad U_0^S = \frac{\delta V^b(1)}{1 + \delta} \ . \tag{1}$$

I will denote by $P^b(X) = V^b(X)/(1 + \delta)$ the seller's (accepted) price offer on the portion X under complete information. The argument of $P^b(X)$ will be suppressed whenever X is transparent.

[2] In the present model, the buyer determines his consumption path optimally given prospective agreements, which are in turn affected by the buyer's consumption path and bargaining history. The consumption decision is not explicitly treated as a strategy in the bargaining game (which would require that the buyer's consumption be observable to the seller) in order to keep the analysis tractable.

[3] The optimal consumption path $\{c_0^b, c_1^b, \dots\}$ obviously depends on the pie stock X of the buyer. We choose not to represent the pie stock X as an argument of c_t^b to economize on notation.

The equilibrium concept for the incomplete information game is sequential equilibrium (see Rubinstein (1985) for a formal definition in the alternating offers bargaining context). The following preliminary result can be proved using the arguments in Lemmata 3.1 and 3.2 in Grossmann and Perry (1986).

Lemma 2.1. *In any sequential equilibrium, (i) B's payoff is at least* $\max\{0, V^b(1) - V^H(1)/(1+\delta)\}$*, at most* $V^b(1) - \delta V^L(1)/(1+\delta)$*; (ii) S's payoff discounted to the date at which she receives the first offer is at least* $\delta V^L(1)/(1+\delta)$*; (iii) given a pie of size $X \le 1$ that S retains, S always accepts the offer* (P, X) *if* $P \ge \delta V^H(X)/(1+\delta)$*. Moreover, S never delays her offer.*

I adopt the refinement on beliefs off the equilibrium path in Admati and Perry (1987).[4] A sequential equilibrium that satisfies (A2) and the restriction below is hereafter called "equilibrium".

Refinement (R): Fix a sequential equilibrium path and a history h^N after which the buyer is active. Consider a "deviant" delay $\hat{\Gamma}_{N+1}$ followed by the offer $(\hat{P}_{N+1}, \hat{X}_{N+1})$. Call it a "bad" deviation for the buyer of type b if the best continuation payoff he can so obtain (no matter S's beliefs after this new history) is lower than his continuation equilibrium payoff. Suppose that the deviation in question is not "bad"for the type-$\underline{b}$ buyer. Then, the seller's belief must put zero probability on b after the history $h^{N+1} = h^N \times \{\hat{\Gamma}_{N+1}, (\hat{P}_{N+1}, \hat{X}_{N+1})\}$.

I close this section with the definition of a "continuation equilibrium price $\overline{\overline{P}}^L(X, Z_N)$" offered after a history h^N of N bargaining rounds where $\pi(h^N) = 0$, the buyer retains the portion Z_N and the seller has the portion $X > 0$ at the beginning of the current consumption period dated τN[5] Suppose that S is the active player and let t_N denote the time, with $t_N > \tau N$. If $Z_N = 0$, the arguments in Lemma 2.2 of Admati and Perry (1987) can be applied to show that in equilibrium S offers the price $V^L(X)/(1+\delta)$ on X and B_L accepts.[6] For $Z_N > 0$, let $\{c^L_{\tau N}, c^L_{\tau N+1}, \ldots\}$ denote the optimal consumption path that solves Problem MAX where the pie size is Z_N. B_L will deplete his pie stock Z_N at some future date $T(Z_N)$ if no agreement is reached between dates t_N and $T(Z_N)$. Suppose this is the case and consider the continuation game extending from bargaining time $t \in (T(Z_N), T(Z_N)+1]$. S retains the portion X whereas B_L has none. If B_L is active, he offers $P = \delta V^L(X)/(1+\delta)$, if S is active, she offers $\delta^{T(Z_N)+1-t} V^L(X)/(1+\delta)$. Both offers are accepted. Moving backwards in time, it can be shown that there is a unique continuation equilibrium price that S offers after history h^N where $\pi(h^N) = 0$, which is accepted by B_L. Let $\overline{\overline{P}}^L(X, Z_N)$ denote this price. Note that if $Z_N = 0$, bargaining rounds and consumption periods are

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[4] See, e.g., Rubinstein (1985) and Kreps (1990) on such refinements.

[5] The price $\overline{\overline{P}}^L(X, Z_N)$ is not offered in a first-round separating equilibrium. It is a price offered off this equilibrium path, when the strong buyer or the seller deviates.

[6] Note that in the complete information version of this game, the continuation game just mentioned would be a Rubinstein bargaining game with a pie of size X if $c^L_0 = X$, that is, if the buyer optimally consumes the portion X immediately. In this case, an accepted offer on X ends the game whereas rejections generate repetitions of structurally identical bargaining rounds. The price $V^L(X)/(1+\delta)$ is the corresponding SPE price offered by S.

then synchronized and $\overset{=}{P}^L(X, 0) = V^L(X)/(1+\delta)$. I will use the shorthand notation $\overset{=}{P}^L$ for $\overset{=}{P}^L(X, Z)$ when the arguments X, Z are transparent. For $\pi(h^N) = 1$, one can similarly define $\overset{=}{P}^H(X, Z_N)$.

3 First-round separating equilibria: the case of a consumption good

A first-round separating equilibrium (FRSE) is an equilibrium in which the seller updates her prior assessment π_{-1} to either $\pi_0 = 1$ or $\pi_0 = 0$ once she receives the first (possibly delayed and restricted) offer (P_0, X). If beliefs are revised to $\pi_0 = 0$ after an offer involving $X < 1$, the next round S offers $(P_1, 1 - X)$ without delay, which B_L accepts and the game ends. B_H makes his complete information offer on the whole pie and S accepts. I allow S to hold optimistic beliefs. Under such beliefs, whenever S receives an offer that B_H can potentially imitate instead of making his complete information offer $(\delta P^H, 1)$ at time zero, S puts probability one on B_H. It is easy to establish that B_H's first-round price offer must be his complete-information offer δP^H. The analysis focuses on the equilibrium behavior of S and B_L. The following condition, coupled with (A1), is necessary for B_L's efficient signaling strategy to always involve a restricted offer.

(A3) The first-period optimal consumptions from $X = 1$ satisfy $c_0^H > c_0^L$.

Let $\succ_S, \succ_L, \succ_H$ denote the preferences of S, L and H over bargaining outcomes. The strategies forming a FRSE must satisfy three conditions.

$$\text{(C1)} \qquad \{0, (\delta P^H, 1)^A\} \succeq_H H\{\Gamma, (P_0, X)^A\} \wedge \{1 + \Gamma, (P_1, 1 - X)^A\} .$$

Condition (C1) defines, for B_H, the set of restricted offers (P_0, X) with delay Γ followed one period later by S's offer $(P_1, 1 - X)$ that are at most as good as the immediate (complete information) outcome $\{0, (\delta P^H, 1)^A\}$. Replacing "most" by "least" and B_H by B_L in the above sentence yields (C2) for B_L:

$$\text{(C2)} \qquad \{0, (\delta P^H, 1)^A\} \prec_L \{\Gamma, (P_0, X)^A\} \wedge \{1 + \Gamma, (P_1, 1 - X)^A\} .$$

In a FRSE S prefers accepting the restricted offer (P_0, X) and offering P_1 on $1 - X$, to rejecting (P_0, X) and making the accepted offer $(P^L, 1)$ the next round:

$$\text{(C3)} \qquad \{0, (P_0, X)^A\} \wedge \{1, (P_1, 1 - X)^A\} \succeq_S \{1, (P^L, 1)^A\} .$$

B_L accepts the price $P^L = V^L(1)/(1 + \delta)$ on the whole pie by Lemma 2.1. The prices P_0 and P_1 are related as follows. Given B_L's first-period consumption c_0^L from $X = 1$, let $Z = \max\{0, X - c_0^L\}$ be the portion that B_L leaves to the next consumption period, in anticipation of the seller's updated belief $\pi = 0$ and the agreement coming over $1 - X$. Then the price offered by S on the portion $1 - X$ is $P_1 = \overset{=}{P}^L(1 - X, Z)$, which is decreasing in X, approaching zero as $X \to 1$. As $X \to c_0^L$ from above, Z approaches zero, and therefore $\overset{=}{P}^L(1 - X, Z)$ approaches

$P^L(1-X) = V^L(1-X)/(1+\delta)$. Along the equilibrium path, S should therefore accept the offer (P_0, X) if $P_0 + \delta \overline{\overline{P}}^L(1-X, Z) \geq \delta V^L(1)/(1+\delta) \equiv \delta P^L$. Defining the price $\overline{P}_X \equiv \delta[V^L(1)/(1+\delta) - \overline{\overline{P}}^L(1-X, Z)]$ for the portion X, Condition (C3) can be expressed as $P_0 \geq \overline{P}_X$. Since $\overline{\overline{P}}^L(1-X, Z)$ is decreasing in X, the price $\overline{P}_X$ defined above must be increasing in X. Now define the"“no-imitation” region through the set $NI(P_0, X, \Gamma) = \{(P_0, X, \Gamma)$ satisfies conditions (C1), (C2) and (C3)$\}$. If the seller receives an offer (P_0, X) after delay Γ such that $(P_0, X, \Gamma) \in NI(P_0, X, \Gamma)$, Refinement R stipulates that she revise her prior assessment to $\pi = 0$. Let $NI^*(P_0, X, \Gamma)$ denote the set of (possibly restricted) offers and delays that maximize B_L's payoff in the no-imitation region $NI(P_0, X, \Gamma)$.[7]

Lemma 3.1. *For any* $(P_0^*, X^*, \Gamma^*) \in NI^*(P_0, X, \Gamma)$*, (C1) and (C3) hold with indifference.*

It will be useful to define a restricted offer $\widetilde{XH}$, through $(\overline{P}_{\widetilde{XH}}, \widetilde{XH}, 0)$ and $(\overline{P}^L, 1-\widetilde{XH}, 1) \sim_H (\delta P^H, 1, 0)$. At the restricted offer $\widetilde{XH}$, B_H is indifferent between the immediate payoff $V^H(1)/(1+\delta)$ and paying the price $P_0 = \overline{P}_{\widetilde{XH}}$ for $X = \widetilde{XH}$ to imitate B_L. Therefore, by Lemma 3.1, it is possible to signal a low valuation through a pure restricted offer by choosing $X \leq \widetilde{XH}$. The following lemma locates $\widetilde{XH}$.

Lemma 3.2. $\widetilde{XH} \in (0, c_0^H)$.

Given an offer on the portion $X \in [\widetilde{XH}, 1]$ and the price $P_0 = \overline{P}_X$, define $\Gamma = \Gamma(X)$ as the delay such that Condition (C1) holds with indifference. For $X < \widetilde{XH}$, set $\Gamma(X) = 0$.

Proposition 3.1. *Under (A1), (A2) and (A3),* B_L*'s FRSE strategy does not involve pure delay. Furthermore, if* $c_0^L \leq \widetilde{XH}$*,* B_L*'s FRSE strategy consists of a pure restricted offer. In this case,* B_L costlessly *signals his type.*

Proof. Suppose that B_L's FRSE offer (P_0^*, X^*) involves $X^* \in [c_0^H, 1)$ after delay $\Gamma(X^*)$. Since the consumption paths are not distorted, $\Gamma(X^*) = \Gamma(1)$ and B_L's payoff is $\delta^{\Gamma(1)} V^L(1)/(1+\delta)$. Since B_L would also obtain this payoff by making his complete information offer $\delta V^L(1)/(1+\delta)$ after pure delay $\Gamma(1)$, the resulting bargaining outcome would be shorter, which violates (A2). Therefore $X^* \in [c_0^H, 1]$ cannot be an equilibrium strategy.

Consider the case $\widetilde{XH} < c_0^L$, and let $X \in [c_0^L, c_0^H)$. The bargaining outcome if $b = L$ is $\rho = \{\Gamma(X), (P_0, X)^A\} \wedge \{1 + \Gamma(X), (\overline{\overline{P}}^L, 1-X)^A\}$ where $(P_0, X, \Gamma(X)) \in NI(P_0, X, \Gamma)$. By Lemma 2.1, S accepts the price $P_0 = \overline{P}_X \equiv \delta V^L(1)/(1+\delta) - \delta \overline{\overline{P}}^L$ and (C3) holds with indifference in accordance with Lemma 3.1. Note that $u_H(X) + \delta V^H(1-X) < V^H(1)$, and $u_H(X) + \delta V^H(1-X)$ is increasing in X for $X < c_0^H$. Since we are in the case $\widetilde{XH} < c_0^L$ and $X \in [c_0^L, c_0^H)$ is assumed,

[7] The omitted proofs can be found in the working paper, Bac (1997).

B_H must prefer the outcome ρ where $X \in [c_0^L, c_0^H)$ and $\Gamma = 0$ to his complete information outcome:

$$u_H(X) + V^H(1-X) - \frac{\delta V^L(1)}{1+\delta} > \frac{V^H(1)}{1+\delta} .$$

That is, $\Gamma(X)$ must be strictly positive to satisfy (C1) with equality. Note that $\Gamma(X)$ is increasing (hence, U_0^{BL} is decreasing) in X for $X \in [c_0^L, c_0^H)$, and $\Gamma(c_0^H) = \Gamma(1)$. Thus any offer restriction X from the range $[c_0^L, c_0^H)$ yields B_L a payoff that exceeds $\delta^{\Gamma(1)} V^L(1)/(1+\delta)$, and a pure delay is never used if $\widetilde{XH} < c_0^L$.

Consider now the case $\widetilde{XH} \geq c_0^L$. Then, $\Gamma^* = 0$ for any $(P_0^*, X^*, \Gamma^*) \in NI^*(P_0, X, \Gamma)$. To see this, recall that $\{0, (\bar{P}_{\widetilde{XH}}, \widetilde{XH})^A\} \wedge \{1, (\bar{\bar{P}}^L, 1 - \widetilde{XH})^A\} \sim_H \{0, (\delta P^H, 1)^A\}$ by definition of $\widetilde{XH}$, but because $\widetilde{XH} \geq c_0^L$ is assumed, the following must hold: $\{0, (\bar{P}_{\widetilde{XH}}, \widetilde{XH})^A\} \wedge \{1, (\bar{\bar{P}}^L, 1 - \widetilde{XH})^A\} \sim_L \{0, (\delta P^L, 1)^A\} \succ_L \{0, (\delta P^H, 1)^A\}$. Note that $\bar{P}_{\widetilde{XH}} + \delta \bar{\bar{P}}^L = \delta V^L(1)/(1+\delta)$, hence (C1), (C2) and (C3) are all satisfied. Clearly, any combination of restricted offer and positive delay that satisfies these conditions would decrease U_0^{BL} below $V^L(1)/(1+\delta)$. Therefore no delay is used, $\Gamma^* = 0$, if $\widetilde{XH} \geq c_0^L$. qed.

In equilibrium B_L signals his bargaining power at zero cost and delay, through a pure restricted offer $X^* \in [c_0^L, \widetilde{XH}]$ if $\widetilde{XH} \geq c_0^L$, that is, if the buyer types are sufficiently heterogeneous so that B_L's optimal consumption of the pie is sufficiently more "conservative" than B_H's. The condition $c_0^L < c_0^H$ stated in (A3) is not enough for costless signaling; c_0^L must be sufficiently lower than c_0^H, at most equal to $\widetilde{XH}$. This requires that $u_L(.)$ be sufficiently more concave than $u_H(.)$. I focus below on the case $\widetilde{XH} < c_0^L$ to characterize B_L's equilibrium restricted offer X^* and discuss the role of assumption (A3) (which implies, but is not implied by, strict concavity of $u_L(.)$). Dropping (A3) means allowing for $c_0^L \geq c_0^H$. It is possible to find preference structures for buyer types that satisfy (A1) but fail (A3), such that a pure delay $\Gamma(1)$ is the best signaling strategy of B_L. The following proposition provides a condition in terms of per-period utilities of buyer types; no reference is made to relative consumption paths.

Proposition 3.2. *A pure delay $\Gamma(1)$ followed by a comprehensive offer is B_L's FRSE strategy if and only if, for all X in $(0,1)$,*

$$\frac{V^L(1)/(1+\delta)}{V^H(1) - \delta V^L(1)/(1+\delta)} \geq \frac{u_L(X) + \delta V^L(1-X) - \delta V^L(1)/(1+\delta)}{u_H(X) + \delta V^H(1-X) - \delta V^L(1)/(1+\delta)} . \tag{2}$$

The left-hand side of (2) is the ratio of the two buyer types' payoffs viewed from time $\Gamma(1)$, while the right-hand side is the same payoff ratio viewed from time $\Gamma(X)$. The optimal signaling mode is therefore determined by a relative evaluation of payoffs: pure delay will be used if and only if B_L cannot find an offer restriction $X < 1$ such that his payoff viewed from date $\Gamma(X)$ decreases relatively less than B_H's payoff, with respect to the payoffs viewed from time $\Gamma(1)$ after pure delay. Condition (2) fails automatically if B_L's optimal consumption path is more conservative than B_H's, i.e., if $c_0^H > c_0^L$, as assumed in (A3). This is so because for any $X \in [c_0^L, c_0^H)$ B_L's consumption is not distorted ($u_L(X) + \delta V^L(1-X) = V^L(1)$) while B_H's is distorted ($u_H(X) + \delta V^H(1-X) < V^H(1)$). Condition (2) may hold if $c_0^L \geq c_0^H$. Therefore, if (A1) holds, so must (A3) for B_L's signaling strategy to involve an offer restriction with probability one.[8]

4 Effective signaling in bargaining over durable goods

Consider now a durable good that promises a constant stream of benefits to the buyer, denoted $u_b(X)$ per period. I assume that $u_b(X)$ is a strictly increasing in X, $u_H(X) > u_L(X)$ for all $X \in (0,1]$, and that the good does not depreciate. The buyer's payoff from the first agreement $(P, X)^A$ reached at time t is $\delta^t[W^b(X) - P]$, where $W^b(X) = u_b(X)/(1-\delta)$ is his total discounted utility from, or valuation of, the portion X. Note that $u_H(X) > u_L(X)$ implies $W^H(X) > W^L(X)$, which means that B_H has a higher valuation than B_L. The buyer's payoff can be defined by appropriately discounting the payoffs associated with each agreement. The seller's payoff is simply the discounted value of payments.

Under complete information, in the SPE of this bargaining game where S retains the portion $1-X$, S offers the price $\overline{\overline{P}}^b(X) \equiv [W^b(1) - W^b(X)]/(1+\delta)$ and B offers the price $\delta\overline{\overline{P}}^b(X)$. Both offers are made without delay and are accepted. The SPE payoffs of this game are as given by (1), where $W^b(1)$ replaces $V^b(1)$.

The analysis of FRSE proceeds as in Sect. 3; the details are omitted. B_H makes his complete information offer $\delta P^H = \delta W^H(1)/(1+\delta)$ and S accepts, B_L delays his offer for Γ units of time and offers P_0 on the portion X. S accepts and offers P_1 at time $\Gamma + 1$ on $1 - X$, which is accepted by B_L. The conditions (C4), (C5) and (C6), expressed below in terms of discounted payoffs, are the counterparts of conditions (C1), (C2) and (C3).

$$\text{(C4)} \qquad \frac{W^H(1)}{1+\delta} \geq \delta^{\Gamma}[u_H(X) + \delta W^H(1) - P_0 - \delta P_1] \,,$$

$$\text{(C5)} \qquad W^L(1) - \frac{\delta W^H(1)}{1+\delta} \leq \delta^{\Gamma}\left[u_L(X) + \delta W^L(1) - \frac{\delta W^L(1)}{1+\delta}\right] ,$$

$$\text{(C6)} \qquad P_0 + \delta P_1 \geq \delta W^L(1)/(1+\delta) \,.$$

With minor modifications, the results in Lemmata 2.1 and 3.1 apply to the case of a durable good. Define the price $\overline{P}_X = \delta W^L(X)/(1+\delta)$. Note that $\overline{P}_X + \delta\overline{\overline{P}}^L(X) =$

[8] A simple observation that follows from Proposition 3.2 is that the standard case of linear preferences $u_b = bX$ where $b = L, H$ and $L < H$ satisfy (2). Thus, pure delay is the dominating signaling mode of B_L in the case of linear preferences. There is no signaling motivation for restricted offers if strategic delay is available and preferences over portions of the pie are linear. Note that such preferences generate no motive for consumption smoothing given the entire pie; indeed, $c_0^L = c_0^H = 1$. In this case we obtain the result presented in Bac (1999), where the buyer is assumed to "consume" his purchases immediately.

$\delta W^L(1)/(1+\delta)$. By Lemma 3.1, in a FRSE (C6) must hold with equality, thus $P_0 = \overline{P}_X$ and $P_1 = \overline{\overline{P}}^L(X)$. If $B = B_L$, the first agreement $(\overline{P}_X, X)^A$ is reached at time Γ, and the second, $(\overline{\overline{P}}^L(X), 1-X)^A$, at time $1+\Gamma$. In a FRSE, B_L's restricted offer on X with a corresponding delay $\Gamma(X)$ and price $\overline{P}_X$ maximizes his payoff given at the right hand side of (C5), while holding (C4) and (C6) with equality. Below I present a condition depending on the discount factor and the shapes of $u_L(.)$ and $u_H(.)$ under which B_L uses pure delay.

Proposition 4.1. *B_L's FRSE strategy involves pure delay if and only if, for all $X \in (0,1]$,*

$$\delta\left(1-\frac{u_L(X)}{u_L(1)}\right)\left(1-\frac{u_L(1)}{u_H(1)}\right) > \frac{u_L(X)}{u_L(1)} - \frac{u_H(X)}{u_H(1)} \,. \tag{3}$$

The proof follows using (C4) and (C6) to solve for $\delta^{\Gamma(X)}$ and evaluating B_L's utility at $X = 1$. The left hand side of (3) is always positive. Thus, if

$$\frac{u_H(1)}{u_L(1)} \leq \frac{u_H(X)}{u_L(X)} \quad \text{for all} \quad X \in (0,1] \,, \tag{4}$$

then B_L should signal his type through pure delay. This would be the case if, for instance, the utility ratio $u_H(X)/u_L(X)$ is decreasing in X. Condition (4) holds when u_H is a continuous, strictly concave transformation $f : R \to R$ of u_L such that $u_H = f(u_L) = A[u_L]^z$ where $z < 1$ and A is sufficiently large to guarantee $u_H(X) > u_L(X)$ for all X. In this case (4) becomes $[u_L(1)]^{z-1} \leq [u_L(X)]^{z-1}$, which holds because $z < 1$. Condition (4) also holds if u_H is a linear transformation of u_L of the form $u_H = au_L$ with $a > 1$. Thus, *for a large class of (concave- or linear-) affiliated buyer types, efficient signaling of bargaining power takes the form of pure delay*. On the other hand, if u_H is a strictly convex transform of u_L in the form $u_H = [u_L]^z$ with $z > 1$, (4) will fail but (3) may still hold.

Proposition 4.2 provides two conditions under which a restricted offer is used. For a portion X in the left neighborhood of one, define $\epsilon_L(X) = u_L(1) - u_L(X)$ and $\epsilon_H(X) = u_H(1) - u_H(X)$. B_L is said to be *relatively satiated* if $\epsilon_L(X) < \epsilon_H(X)u_L(1)/u_H(1)$. B_L is said to be *relatively and sufficiently satiated* at large X if, in addition, $\epsilon_L(X)$ is sufficiently close to zero (so that the inequality in (3) is reversed).

Proposition 4.2. *If (i) B_L is relatively and sufficiently satiated at $X = 1$, or (ii) B_L is relatively satiated at $X = 1$ and $\delta < \delta^C(\epsilon_L)$ where $\delta^C(\epsilon_L)$ is a critical level of the discount factor, then B_L's FRSE strategy involves a restricted offer.*

The condition $\epsilon_L(X) < \epsilon_H(X)u_L(1)/u_H(1)$ guarantees that the right hand side of (3) is positive, and $\epsilon_L(X)$ sufficiently close to zero implies that the left hand side of (3) is almost zero. Basically, if B_L is relatively and sufficiently satiated, his marginal discounted utility of the durable good vanishes as $X \to 1$. This is sufficient but not necessary for B_L's best signaling strategy to involve a restricted offer. If B_L is only relatively satiated at $X = 1$ so that $\epsilon_L(X) < \epsilon_H(X)u_L(1)/u_H(1)$ and $\epsilon_L(X)$ is bounded away from zero as $X \to 1$, then the left hand side of (3) will exceed the right hand side for δ sufficiently close to one. There exists a critical level of the discount factor $\delta^C(\epsilon_L)$ such that (3) becomes an equality, and the inequality in (3) is reversed if $\delta < \delta^C(\epsilon_L)$. For such low discount factors, B_L's first-round separating equilibrium strategy will involve some restricted offer even though his marginal discounted utility at $X = 1$ is bounded away from zero, provided that he is only *relatively* satiated at $X = 1$.

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